Power Flow Modeling Reference Document

Version 1.1.1 (latest revision was June 2013)

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Table of Contents

Revisions

Date	Version	Notes
7/1/2012	1.0.0	
8/18/2012	1.0.1	Minor formatting changes. Added references to other Forum documents Appendix 2 – Replaced diagram and added reference to Generator Specifications Reference Document
6/2013	1.1.1	

Introduction

The Audience

The Forum's Modeling Practices Group developed this document for the following audience:

- Transmission planners and operations planners
- Engineers who validate and provide generation models to the planning engineers
- Operating engineers who maintain real-time models for state estimators and energy management systems
- Anyone with an interest in powerflow modeling concepts

This reference document is a companion to the Transmission Forum's "**Power Flow Guidelines**," and covers the terms and technical details that the transmission planner or generation engineer uses to develop powerflow models. This is not intended to be a rigorous explanation of ac powerflow concepts, but a simple overview that can lead the reader to pursue further discussions with, and explanations from, other experts within the utility.

Section 1.   The Power Flow Model

Why the power flow model is important

The power flow model is a case study—a "snapshot"—that provides the planner two general kinds of information based on a particular transmission, generation, and load configuration at some specific point in time:

1. **Voltages and angles at each bus** relative to the swing bus, which are listed in per-unit, and
2. **Real and reactive flows through each branch** (element and transmission line) between the busses, listed in MW and MVAr.

If the planner finds the bus voltages, angles, and real and reactive flows are within acceptable levels, then we can reasonably assume the system will remain stable and within operating limits for that particular configuration. However, if the voltages or power flows exceed operating limits, then we know that particular transmission, generation, and load configuration could lead to unreliable operation, such as overloaded equipment or a voltage collapse. And, if the powerflow case fails to solve, we can assume the transmission system would not have remained interconnected under those conditions.

How the powerflow model works

Solving the powerflow case means finding the voltage and phase angle at every bus in the simulation. Each bus is represented by an algebraic equation of the form:

$$P_k = \sum_{j=1}^{N} |V_k||V_j|\left(G_{kj}\cos(\theta_k-\theta_j)+B_{kj}\sin(\theta_k-\theta_j)\right)$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j|\left(G_{kj}\sin(\theta_k-\theta_j)-B_{kj}\cos(\theta_k-\theta_j)\right)$$

Figure 1 - Transmission line model using Y-bus representation

Where the known values are:

P_k **and** Q_k. These are the real and reactive power, respectively, injected into bus k. For a load bus, these are the real and reactive loads. For a generator bus, these are the real and reactive generator outputs, which we call **PMAX**, **QMAX**, **PMIN**, and **QMIN.**

G_{kj} **and** B_{kj}. These are the real and reactive admittances (simply the inverse of the impedances), respectively, between buses K and J.

And the unknowns we're solving for are:

V_k **and** V_j. These are the voltages at bus k and j, respectively, and

θ_k **and** θ_j. These are the phase angles, respectively, at busses K and J compared to the swing bus. (These are not power factor angles.)

Therefore, the powerflow model is the solution of as many simultaneous equations as there are busses in the model.

Power flow solutions versus stability analysis. A Power flow, or steady state, model represents a snapshot of the system and the simulation results only represent the flows of the specific model assuming that the system is stable and operating at 60 Hz. The stability (or "dynamic") analysis simulates over seconds or minutes the response of the transmission system (generator rotor angles, voltages, and power flows) to perturbations such as short circuits and sudden generator or load trips and determines if the system will tend to remain stable afterward. Dynamic analysis models are covered in a separate document.

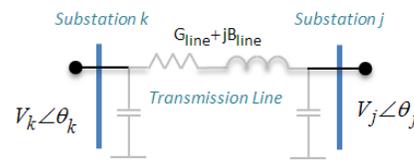**Z-bus versus Y-bus**. Power flow software converts the Z-bus impedances to Y-bus admittances. This fills the Jacobian matrix mostly with zeros, which is the admittance between two unconnected busses.

Phase angle. Some conventions use **δ** to designate the phase angle.

Solution techniques

A variety of solution techniques can be used to solve the equations necessary to calculate voltage and angle for each bus. However, large power flows generally apply iterative solution techniques such as full Newton-Raphson or De-coupled solutions technique. De-coupled solution techniques solve the active and reactive power flows separately by taking advantage of the strong relationship of real power flow to voltage angle difference and reactive power flow to voltage magnitude difference as compared to the weaker relationship of real power to voltage magnitude and reactive power to voltage angle. The solutions are also designed to take advantage of the sparseness of the nodal equations developed to represent the grid (admittances are used in a Jacobian matrix). Over the years, the basic solution technique has evolved and improved until current algorithms can solve extremely large networks, representing thousands of busses, very quickly. Once voltages and angles are known, the power flow then uses basic electrical relationships to calculate flows through lines, transformers, etc. (otherwise known as branches in electrical theory) given the impedances (or admittances) of the various branches.

Note that some representations use low-Z components as a mechanism to calculate flows through near zero impedance devices such as bus-work or circuit breakers, etc. These types of devices should be used carefully as they can cause numerical problems (e.g.: zero divides).

Details, granularity, and equivalents

Depending upon the study objectives, the planner will need to decide if the model has sufficient detail and whether the generator and transmission parameters are sufficiently accurate. Typically, the area of transmission system the planner is focusing on requires a detailed model of the generating stations with explicit models of generator step-up transformers, auxiliary transformers, and auxiliary and station service loads, etc. In areas on the periphery of the area of interest, the planner can use equivalents that lump together generators, busses, and an assortment of transmission lines.

Model Variations

Because the powerflow model is only one snapshot that represents a particular transmission, generation, and load configuration at some point in time, the planner has to solve dozens of powerflow cases to ensure the system will remain within its steady-state operating limits through time, which are often called **planning and operating horizons**, and for many different **system conditions**.

Planning and operating horizons

The planning and operating horizons are typically categorized as 1.) Planning, 2.) Operations planning, and 3.) Real-time.

Long-term planning study cases represent seasonal peak, shoulder, and valley load periods for each of the next five to ten years and incorporate the equipment additions, retirements, and enhancements that are expected to occur. Generally, all breaker and switch positions are assumed to be in their 'normal position' (either opened or closed). Since a power flow simulation solves for the system flows with these facilities modeled at one specific state, additional simulations will be needed to model these devices in their different states. The planner will also study the effects of equipment failures (contingencies) and planned outages that change the topology of the network.

Operations planning powerflow cases represent the system conditions from the **next hour to the next 12 months.** The results of next-day and next-week studies are provided to the system operators to help them understand the configuration of the transmission system they will be operating. The daily loads and likely unit commitment

patterns (including "must-run" units for reliability purposes) are also provided. Therefore, the operations planning power flow model will not include facilities that will be in service beyond the Operational Planning horizon.

Near-real time powerflow cases use state estimators and contingency analysis studies to simulate the current system in operation. Analysis can be performed on such a case to evaluate the impact of contingencies and other system events and to inform the system operator "what happens if…."

System conditions and "base cases"

To run a single powerflow study, the planner enters into the powerflow model the system conditions that represent the year, season or month, time of day, generation dispatch, load magnitude (real and reactive) and location, and interchange. To prepare a **base case**, the planner assumes that all equipment is in service and the generators are economically dispatched within their limits of **PMAX** and **PMIN**, and interchange levels represent schedules that are known at the time. It's common for the system planner to have perhaps eight base case models for a particular year representing four seasonal peak demands, plus various off-peak loads.

Once the base case is judged acceptable, the planner prepares other powerflow cases by changing the generation dispatch, removing generators and transmission elements to represent the planned maintenance schedules and forced outages, and varying the loads and interchange to reflect different weather conditions, emergency situations, and economic scenarios. The planner then applies these cases to obtain different power flow results under these different system conditions.

Some planners use various software tools to prepare the various generator dispatch scenarios that support different load and interchange patterns. These tools include complex unit commitment algorithms and lists of units that will be called on following the list order.

Equipment ratings

The planner models the transmission system to analyze its operation—power flows and voltages—under normal conditions and credible contingencies. **Therefore, equipment ratings should reflect what can be expected without needing to resort to special intervention by field or plant crews**.

This is more straightforward for "static" transmission equipment such as lines, breakers, switches, and transformers whose normal and emergency, short-term ratings are known and easy to model. Generators, however, are considerably more complex, and their operations have to be "fine-tuned" to meet specific conditions that are usually beyond most modeling programs' capabilities to simulate. **Therefore, generators in the planning model should reflect a conservative rating, even though they may be able to produce additional real or reactive power when manually adjusted**. This also means that the generator ratings for modeling purposes may be different than those provided to the marketplace or as part of interconnection agreements.

> **"Model on request."** Alternatively, rather than maintaining a set of base-case models, the planner may build models as needed using a "model on request."

Section 2.   Busses and Bus Voltages

R, X, B.

Power flow and dynamic analysis use per-unit values rather than actual kV and MVA values. These values are normalized to a specified base power and base voltage. See "Appendix 1. – Per-unit Calculations and Expressions" for an explanation of this very important concept.

Base voltage. The voltage for calculating the per-unit value in the power flow calculation. The base voltage is typically chosen as the nominal voltage of the bus. The base voltage for busses connected to each other by transmission lines or series reactive equipment must be the same (See Figure 2)

Nominal voltage. The operating voltage at which the bus is designed to operate.

Per unit voltage. Powerflow models calculate bus voltages in per-unit rather than kV. Per unit voltage is defined as the bus voltage divided by the base voltage. (See Appendix 1. – Per-unit Calculations and Expressions)
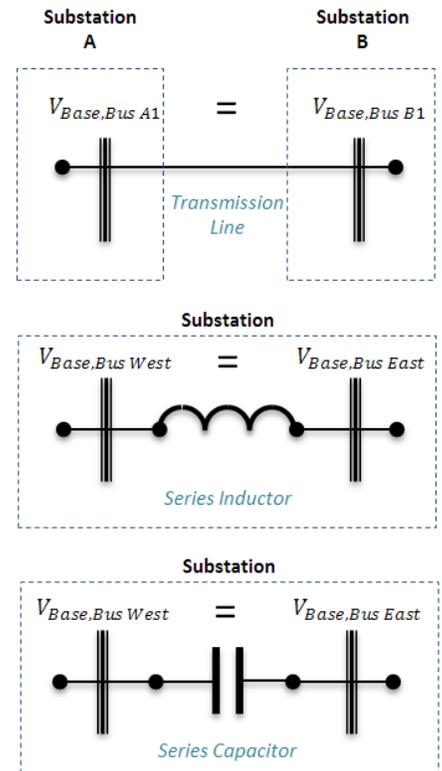

Figure 2 - The base voltage for busses connected by lines or series reactive equipment must be the same.

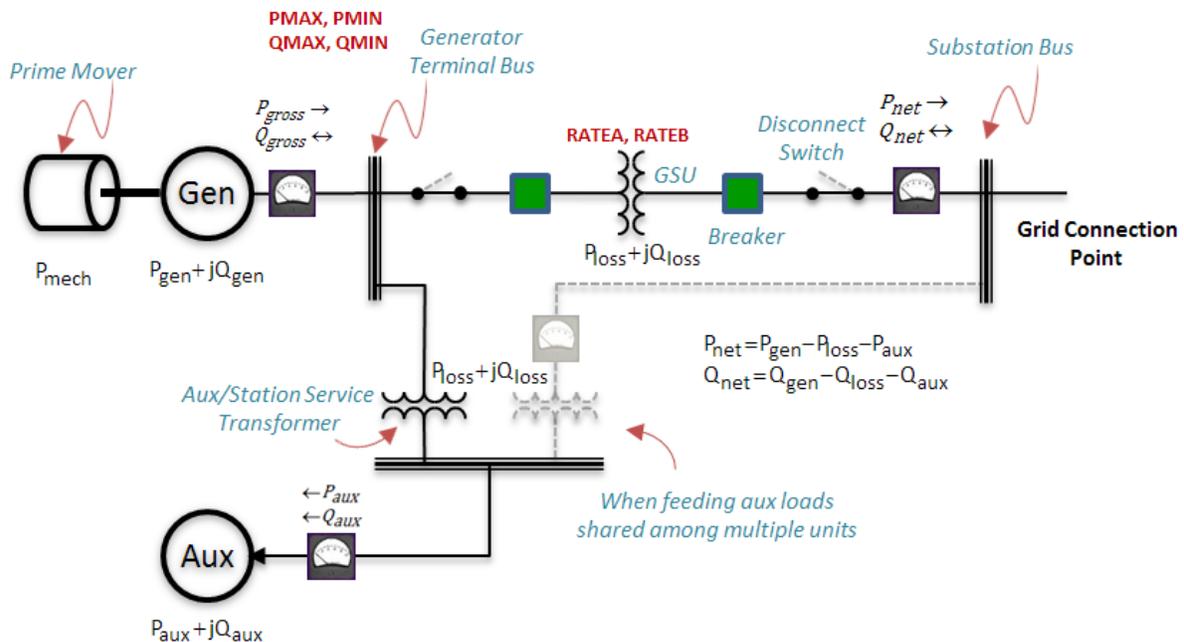# Section 3.   Generators

> **PMAX, QMAX, PMIN, and QMIN.**

Figure 3 - Generating plant model details

The Generating Plant

For this discussion, the generating plant as depicted above comprises the following elements:

Generator

Auxiliary (or station service) generator loads. These include the unit-specific and common plant loads such as pumps, fans, and fuel handling equipment, and is designated as **P_{aux}** and **Q_{aux}**. Auxiliary loads are in the range of 5% of the gross output of the generator.

Auxiliary (or station service) Transformer. A transformer that provides the proper voltage for the auxiliary generator loads. When providing auxiliary power to multiple generators for common services such as fuel handling and condenser pumps, an auxiliary transformer may be connected to the bus on the grid side of the generator switchyard.

Generator. The generator converts Pmech into real and reactive electrical power, Pgen and Qgen.

Gross generation output. The gross output, **P_{gross}** and **Q_{gross}**, are measured at the generator terminals, and are also what the planner uses in the powerflow model to set the generator output, called **PMAX, QMAX, PMIN**, and **QMIN**.

Generator terminal bus. The bus to which the generator leads are connected. The generator auxiliary equipment is often tied to this bus.

Net generation output. The generator's net output is measured on the high-voltage side of the GSU and equals the gross generation less losses from the GSU and auxiliary load.

$$P_{net} = P_{gen} - P_{loss} - P_{aux}$$
$$Q_{net} = Q_{gen} - Q_{loss} - Q_{aux}$$

Prime mover. A steam, hydro, or combustion turbine, or Diesel engine, whose mechanical output is shown as P_{mech}. The mechanical output or a steam or combustion turbine will vary according to the temperature and pressure difference across the turbine. Steam turbine output increases as the condenser water temperature decreases, and combustion turbine output increases as the compressor intake air temperature decreases. Diesel engine output also increases as the intake air decreases. Therefore, the planner uses different values for **PMAX** for summer and winter seasons, and possibly for spring and fall as well.

Hydro turbine output is a function of water supply or river flow requirements and restrictions rather than seasonal temperature. So the planner may use different values of **PMAX** to represent these variations.

Switchyard

Generator step-up (GSU) transformer. The transformer that steps up the generator output voltage (usually 12kV to 24kV) to the grid connection voltage. The GSU normal and emergency rating is designed as **RATEA** and **RATEB**, respectively.

Substation bus. The bus to which the high-side of the GSU(s) are connected.

Switchgear. The transmission equipment between the generator terminal bus and the substation bus, which includes the GSU disconnect switches and breakers, current and potential transformers, and so forth. Switchgear *per se* aren't modeled, but open switches and breakers may imply that equipment is not in service. (See Figure 4)
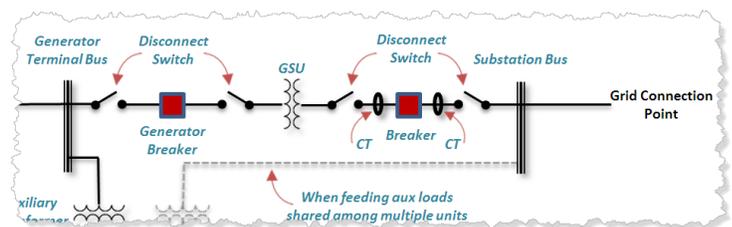

Figure 4 - Switchgear includes disconnect switches and current and potential transformers.

Generator Steady State Characteristics and Operations

The real and reactive power output of a generator are related to each other in a non-linear fashion, and both are bounded by the physical properties of the generator and its dynamic interaction with the rest of the Interconnection. Thus, the model builder (who is usually affiliated with the transmission organization), and generator owner (who may be part of a vertically-integrated organization or independent) must have a common understanding of the factors that drive **PMAX**, **QMAX**, **PMIN**, and **QMIN**.

Generator Capability and the "D-Curve"

In the most basic terms, the generator output is bounded by 1.) The stator current limit, i_{max} ,2.) Rotor current limit during over-excited operation, and 3.) Stator end-winding heating current during underexcited operation. The area within these limits and the Y axis form a "D" shape, hence we call this the generator's D-curve characteristics. (Figure 5).

Generating Plant Capability

The generating plant is further limited by upstream (prime mover) and downstream (plant auxiliary or step-up transformer) equipment mechanical and voltage limits. We can show this by overlaying these limits on the generator D-curve in Figure 6. **However, the powerflow and stability models use the gross generator ratings, which include limits from the prime mover, but account for the plant auxiliary equipment as separate loads**.

Downstream Equipment Limits. Downstream equipment limits include physical limits of the generator or electrical limits or requirements between the generator output terminals and connection point to the grid. For example, it is quite common for station auxiliary voltage to be the limiting factor when evaluating a generating unit's reactive capability and/or its ability to maintain transmission system bus voltage schedule. While the generator itself can operate continuously between 95% and 105% of its nameplate terminal voltage rating, when station auxiliary loads are powered from the generator bus with fixed tap station auxiliary transformers, the generator operating voltage range can be limited by station auxiliary equipment voltage limits. For example, due to auxiliary equipment requirements, a generator's voltage limits can be 97% to 103% of rated terminal voltage instead of 95% to 105%. This is a very common configuration and limitation for traditional steam units. Other types of units (Hydro, CT, Combined Cycle generators) can also have this limitation.

Figure 5 - The operating limits of the generator are bounded by the **rotor current limits** during lagging operation and **stator end winding heating** during leading operation. The result is a "D" curve, which is the shaded part of the graph.

Figure 6 - The plant rating includes the limits of the up- and down-stream equipment, such as the prime mover and down-stream auxiliary equipment or step-up transformer.

Upstream Equipment Limits. Upstream limits include conditions that affect the output of the prime mover. This includes condenser water temperature for steam generators, air temperatures for combustion turbines, and reservoir ("head") heights for hydro generators. Other constraints include boiler limits, governor characteristics, operating points, and environmental regulations.
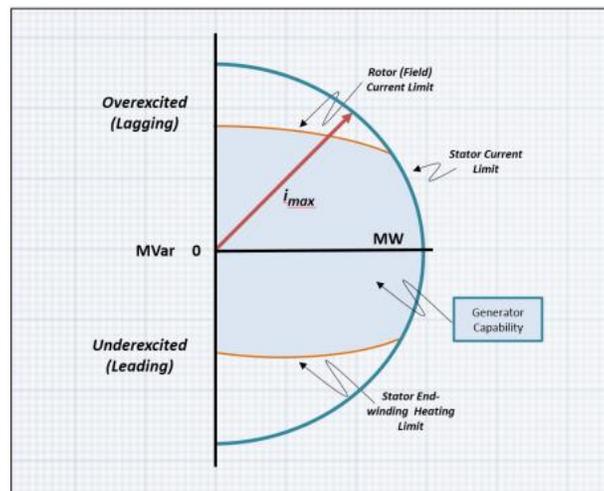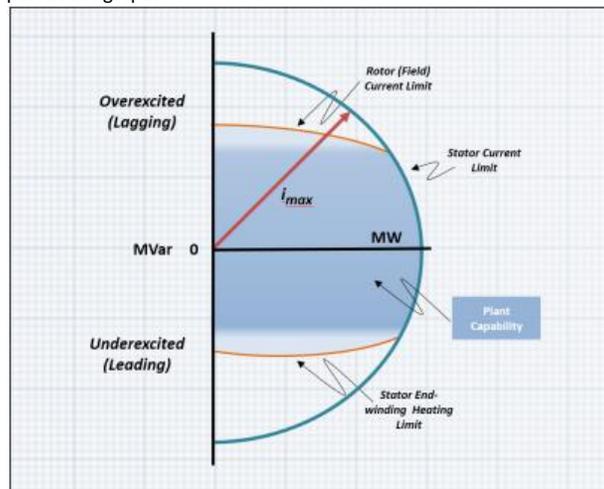
Generator *Limits* Versus *Ratings*

A *limit* and a *rating* are not the same. The generator's real and reactive *limits* are bounded by the rotor and stator currents and stator end-core heating, while the generator's *ratings* are measured as what the generator can actually produce at different ambient and system conditions. The generator operator can change some limits by adjusting the hydrogen cooling pressure, resetting protection trip points, or by plant modifications, and thus incease the unit's ratings. On the other hand, summer cooling water temperatures, temporary prime boiler or turbine limitations, or emission restrictions can reduce the generator's ratings.

Specific Generator Limits, Ratings, and Other Parameters

Figure 7 shows additional limits, ratings, and other paramters on the D-curve.

Figure 7 - D-curve with other limits shown.

Rated Operating Point. The MVA rating of the generator at a particular power factor (90% is typical), and hydrogen pressure.

Power factor. The ratio of the real power (MW) to the apparent power (MVA) that the generator is producing.

Normal Operating Range. The generator's continuous real and reactive power output maneuvering capability over a range of power factors.

Overexcited operation. Operating the generator to produce VARs. This is also called **lagging operation** because the ac current lags the voltage.

Underexcited operation. The generator is absorbing VARs. This is also called **leading operation** because the ac current is leading the voltage. During leading operation, the generator is limited by two things: 1. Loss of synchronism (the steady-state stability limit), and 2. Stator core end heating.

Hydrogen Pressure. The rating of the generator as a function of hydrogen cooling pressure in PSIG[1]. Greater hydrogen pressure provides greater cooling for the generator rotor and stator and allows the machine to operate at a higher output.

Exciter heating. The **rotor's** (exciter's) thermal limit.

Stator end-winding heating. In addition to the electromagnetics of the main **radial** flux distribution across the air gap and in the main body of the stator and rotor, there are end-region effects from the **axial** flux produced. The end-region effects arise from the **end-windings** of the stator and rotor, and core-
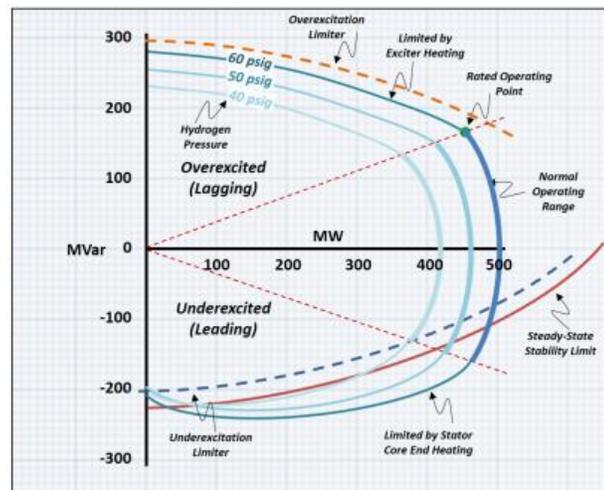

Figure 8 - Generator rotor

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[1] PSIG means pressure relative to atmospheric pressure.

end fringe effects. Both effects vary with the power factor and rotor angle of the generator because of the change in interaction between the stator and rotor magnetic fields. In the **lagging** power factor range, these **two fluxes tend to oppose** and reduce the heating effect. In the **leading** power factor range, the **fluxes tend to sum up** (vector summation) and create higher losses in the core-end.[2]

Other Protection Limits

In addition to these basic physical operating limits, the generator has other protection equipment that limits generator real and reactive output. The figure on the right shows these additional limits on the D curve.
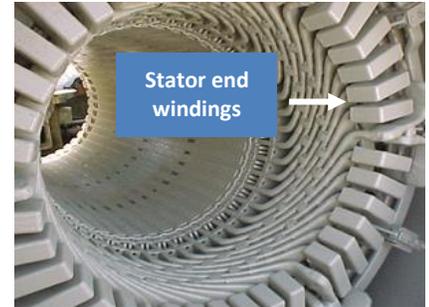
Overexcitation limiter. Limits the rotor's current to prevent thermal damage.

Underexcitation limiter (UEL). Prevents loss of synchronism and stator core end heating when operating at a leading power factor. The UEL is therefore set above the steady-state stability limit.

Steady-state stability limit. The point at which the exciter field is too weak to keep the generator rotor synchronized with the Interconnection. This loss of synchronism can damage the generator and stress the transmission system.

Figure 9 - Generator stator

Generator Model Ratings

The planner needs the **gross** generator ratings **PMIN, PMAX, QMIN, and QMAX from at least four operating points** on the D-curve. **These ratings are interrelated** and shown as the ordered pairs on the graph in Figure 10. In other words, PMAX and PMIN are functions of the reactive output, and QMAX and QMIN are functions of the real power output. The typical powerflow software uses these four ordered pairs to construct a three-segment piecewise linear graph from which the software can interpolate the real and reactive output at the operating points in between. Some powerflow software can use more than four points, which yields more accurate solutions.

PMIN for steam units is typically limited by the **stability and combustion chemistry of the boiler** at low firing rates.[3]

PMAX is limited by the the **boiler, turbine, and generator stator windings**. PMAX is measured at the Rated Operating Point, which usually means at 0.9 power factor. While the D-curve shows that PMAX could be higher if the reactive output was reduced to zero, that's not a realistic operating condition. (See also **Upstream Equipment Limits** below.)

QMAX is limited by the **minimum real power output** and the **thermal limits of the rotor** windings.
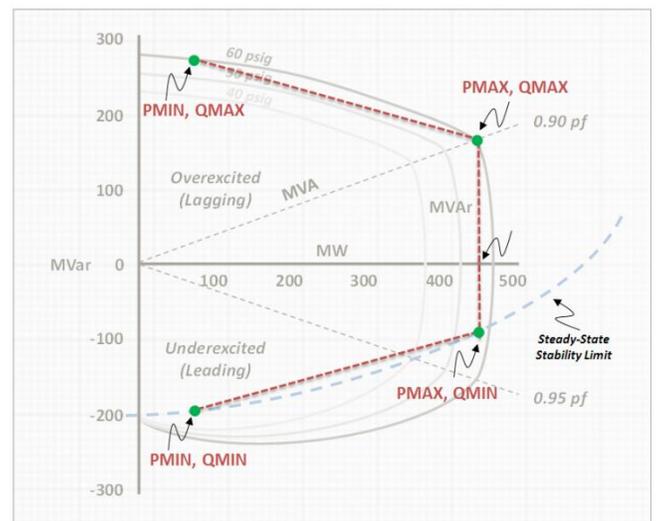

Figure 10 - Piecewise linear representation of the generator limits, PMAX, PMIN, QMAX, QMIN.

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[2] Excerpted and paraphrased from *Operation and Maintenance of Large Turbo-Generators*, © 2004 by IEEE, John Wiley and Sons, Hoboken, NJ.

[3] Combustion turbines, either as peaking units or as part of a base-load combined cycle generator are not normally operated at minimum levels. Hydro turbines per se do not exhibit minimum operating limits. Nuclear units are also base-loaded, but some nuclear units may be cycled.

QMIN is limited by the **minimum real power output** and the **steady-state stability limit**.

Multiple Generator Configurations

The figures below depict two multiple generator configurations—one that connects two generators to a three-winding GSU, and another that uses separate GSUs for each generator. In both configurations, the generators's auxiliaries are supplied by a single auxiliary bus that is connected to the high-side of the station switchyard.
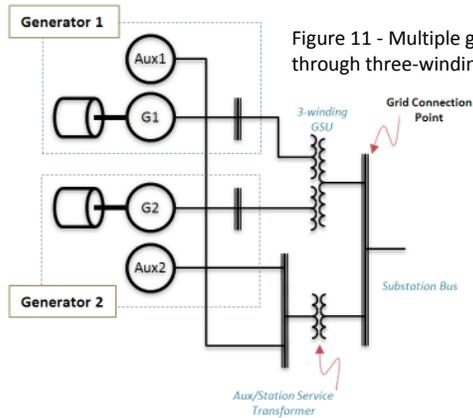

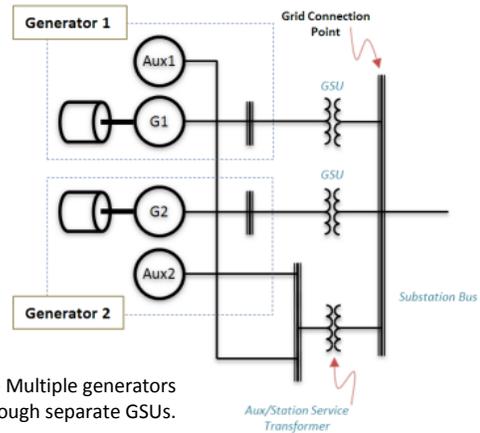Figure 11 - Multiple generators connected through three-winding GSU.

Figure 12 - Multiple generators connected through separate GSUs.

Modeling multiple units

Multiple generating units at the same plant may be combined into an equivalent, single generator if the individual units have the same characteristics. Thus, in Figure 13:

1. Generators of the same size and "D-curve" characteristics that have prime movers of the same size and characteristics may be combined into a single generator with twice the capability.

2. For combined cycle units, the combustion turbines of the same size and type may be combined into a single equivalent combustion turbine generator. Likewise, the steam turbines of the same size and type may be combined into a single equivalent steam turbine generator.
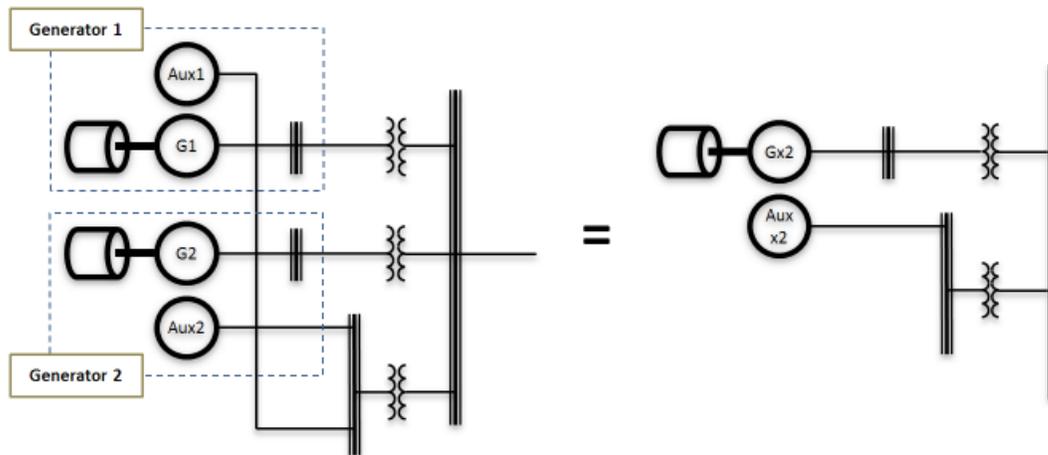

Figure 13 - Generators at the same plant may be combined into a single equivalent generator if the individual units have the same characteristics.

Section 4.   Transmission Lines

For modeling purposes, transmission lines, or "branches," fall into two categories:

1. **Transmission lines** that connect two non-adjacent substations, and
2. **Jumpers** that connect equipment within a substation, or busses between two adjacent substations.

Transmission Lines

Transmission lines are modeled with a resistance (real) component, R_{line}, and a reactive (imaginary) component, X_{line}, comprising an inductive reactance and capacitive reactance distributed along the line. Thus, the impedance of the line is calculated as:

$$\text{Impedance} = Z_{line} = R_{line} + jX_{line}$$

However, as explained in the section, "**How the powerflow model works**," the power flow software converts the bus impedance matrix ("Z-bus") to an admittance matrix ("Y-bus") to take advantage of the sparse bus connection matrix.

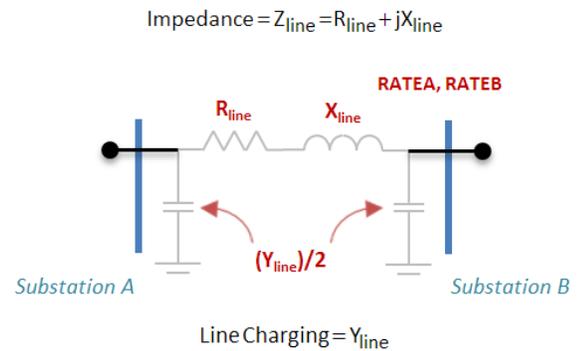$$\text{Impedance} = Z_{line} = R_{line} + jX_{line}$$

Figure 14 - Transmission line model uses impedance values, but converts these to a "Y-bus" matrix using line admittances.

Section 5.   Transformers

The Transformer

Even though they have no moving parts (other than their cooling systems), transformer models are second only to generator models in their complexity. This is partly due to their many different types, configurations, and how they are connected in the transmission system.

Electricity has to be transported from the plant to end-users through the power grid. Transformers are critical elements of the power grid, serving several roles:

Generator step-up transformers (GSUs). At generating stations, transformers, known as GSUs, are used to increase voltage which lowers currents and thus losses as power is transported across the grid. See The Generating Plant for more details.

Figure 15 - Generator step-up and auxiliary transformers

Auxiliary (station service) transformers. Within a power station, auxiliary or station service transformers are used to convert generator output from the generator bus or GSU high-side to supply pumps, fans, lighting, etc. required to run the generators. These transformers and the loads they supply (or at least the load supplied by these transformers modeled at their high-sides) are required to accurately model unit gross and net outputs. Modeling of these transformers may also help support unit testing or transient stability analysis. More discussion of auxiliary and station service transformers can be found in Section 3, "Generators."

Bulk power transformers. Bulk Power transformers serve as interface devices interconnecting different bulk power voltage levels. Typically these large capacity, high voltage devices interface circuits that are 100+ kV. Some are **autotransformers** that share a common winding, while others are traditional, dual winding transformers. (See Figure 16)

Distribution transformers. High voltage transformers are also used to tap into the grid to supply substations which serve distribution feeders supplying pole top, pad-mount, submersible, and other similar equipment and switchgear.

Voltage regulating (tap changing) transformers. Regulating transformers that provide the ability to incrementally adjust transformer turns ratios to control voltage profiles (used on high voltage and supply feeders). **Load tap-changing transformers (LTCs or TCULs)** can have their tap setting changed while under load, while others must be taken off line to adjust their tap settings. (See Figures 17 and 18).

Phase-angle regulating transformers. High voltage, high capacity, Phase Angle Regulators (PARs) provide the ability to alter the phase angle across a transformer to control power flow. (See Figure 19)
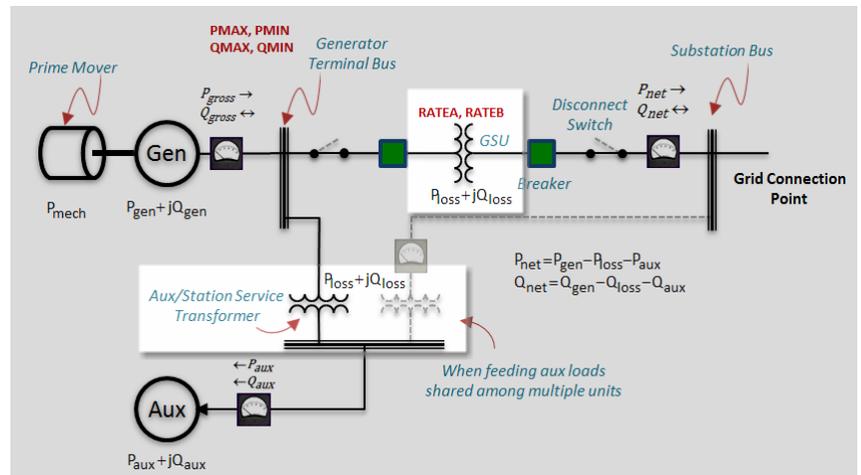
Multiple windings

GSUs and Bulk-power transformers may include a third (tertiary) winding to accommodate multiple generators or supply loads at another voltage.
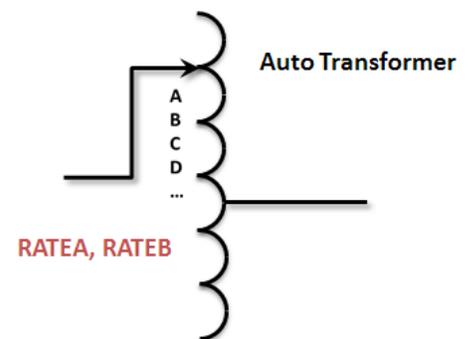

Figure 16 - The autotransformer has a common winding that it tapped to provide the desired secondary voltage. Taps are typically on the high side, but can be on the low side.
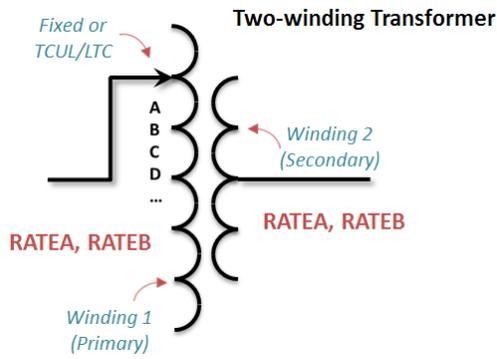
Figure 18 –Two-winding power transformer. This one allows for voltage regulation by adjusting the tap setting. Either side can be tapped, but taps are typically on the high side.
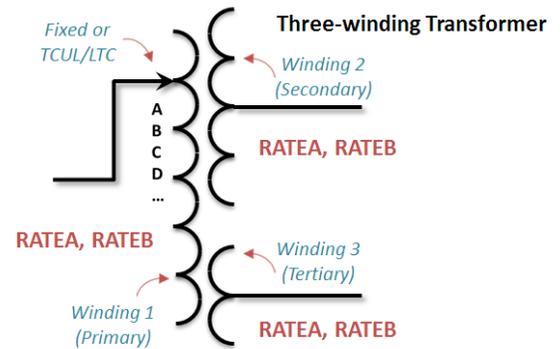
Figure 17 - Three-winding power transformer that can be used to supply loads at two different voltages.

Variable frequency transformers. High voltage, high capacity, variable frequency transformers that provide the ability to control power flow and regulate voltage levels.

Although not usually modeled for analysis of the grid, power systems also employ:

Residential and commercial (end-use) transformers, such as, pole top, pad-mount, submersible and/or vaulted transformers which directly serve customer load, and

Low capacity transformers which serve as transducers for metering or protection circuits (e.g., PTs, CTs).
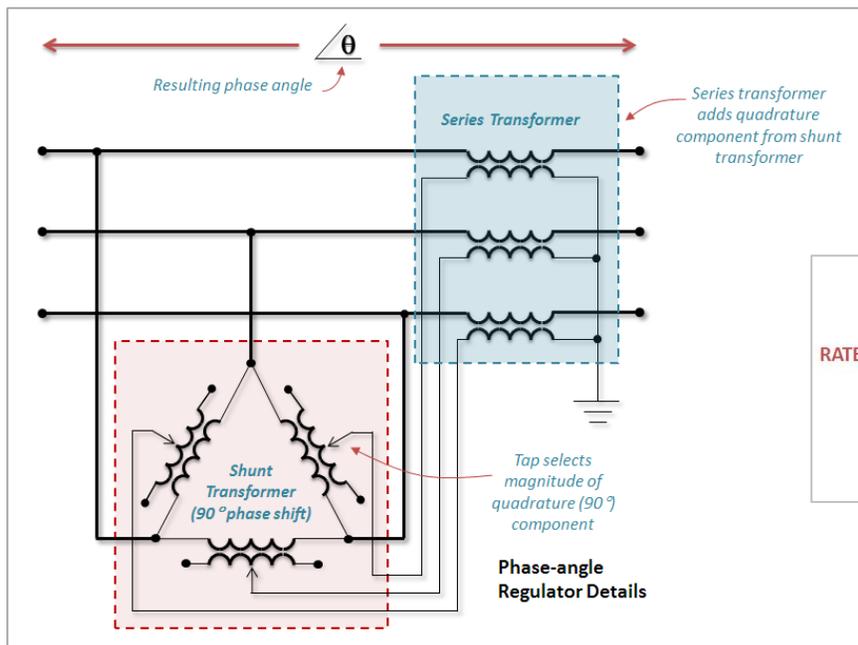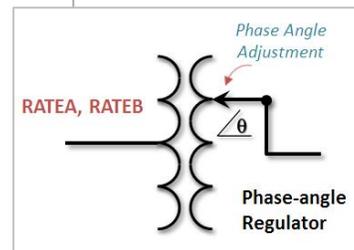

Figure 19 - The phase angle regulator comprises a shunt and series transformer to produce the desired phase angle. The internal details are shown at left, and the model representation below.

Transformer Modifications and Model Updates

As with generators, transformers are expensive, are difficult to replace, and tend to be used and useful over a long time span, possibly exceeding 50 years. Consequently, engineers and planners may be required to adapt equipment to different roles and uses over time which may impact the design of the grid, and therefore, the transformer models.

Transformer Configurations

Several different types of construction techniques are used when manufacturing power transformers capable of carrying large electrical loads. Although single phase transformers are used on the power grid, **many, if not most, transformers are three phase transformers**. The discussions below assume that the system being analyzed is a balanced, three-phase system, which requires a model of only one phase that is referred to as the **positive sequence model.** It is necessary to add details to include negative and zero-sequence models when simulating unbalanced fault conditions.

The most prominent types are two or three winding transformers with a laminated core or shell and auto-transformers. **Laminated steel cores** are generally used in power transformers operated at 50 to 60 hertz as this construction tends to reduce eddy-currents and magnetizing losses. *Power transformer models generally do not represent the magnetizing circuit and associated losses*

The **positive sequence model** assumes that the three phases are balanced, so we can use one phase to model the system. However, when analyzing single- or dual-phase faults, the phase currents will be unbalanced, and a three-phase model must be used to analyze those situations.

How the Transformer is Modeled

The diagram below depicts three ways of modeling a transformer. The **exact model** reflects the in-series resistance, leakage inductance, magnetizing reactance, and core losses from the secondary side to the primary. The **approximate model** is more practical, though a bit less accurate. The **approximate pi model** is a compromise that reflects the series losses to the primary side of the transformer and divides the magnetizing reactance and core losses to both sides.
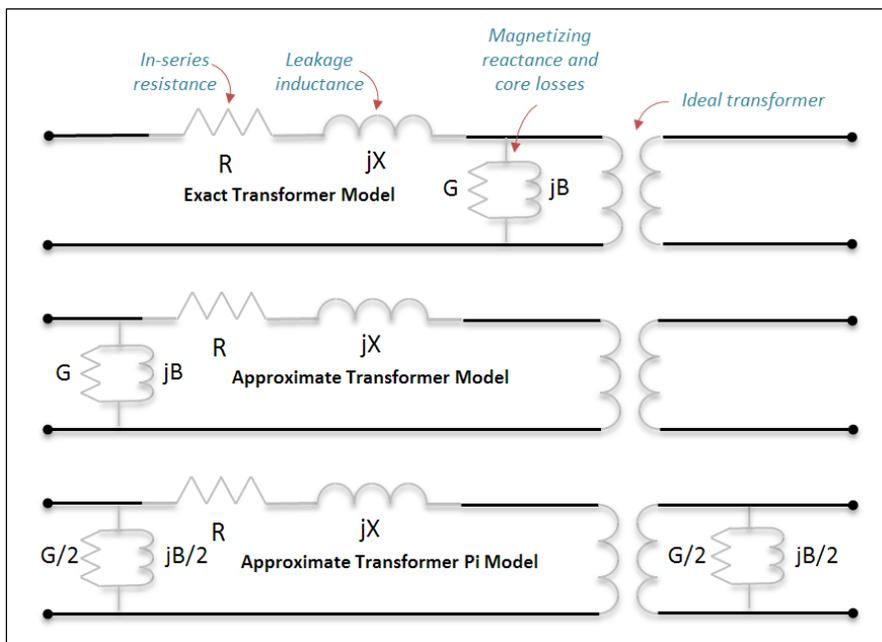

Figure 20 - Transformer equivalent circuit models

Transformer Characteristics and Operations

Basic Limits

Generally, transformer capacity is limited by its ability to dissipate the heat produced by the **internal resistance losses** of the windings and **core heating**, mostly caused by the rapidly changing direction of the magnetic field (hysteresis losses) and eddy currents (eddy current losses). Consider that a 100 MVA transformer operating at 99% efficiency will generate 1 MW of heat that must be removed, so large power transformers will usually incorporate some kind of **active cooling** involving fins, radiators, oil pumps, and fans.
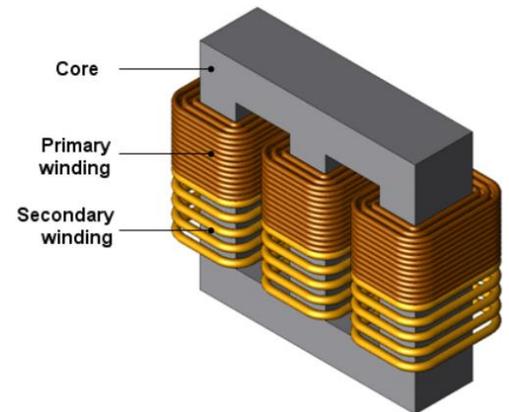
Transformer Model Ratings

Continuous rating. The system model should reflect the **continuous rating** of the transformer; that is, the power that the transformer can handle continuously, which is designated as **RATEA**. The continuous rating, in turn, is a function of the transformer's ability to dissipate its internal heat, which means that the continuous rating can be different when the transformer's active cooling is operating as shown on the transformer nameplate in Figure 21. This transformer has three continuous ratings that are based on the cooling stages that have been activated:
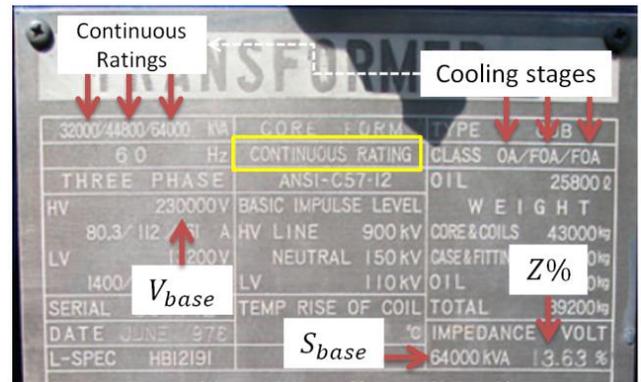

Figure 21 - Transformer nameplate

Continuous Rating	Active Cooling
32 MVA	Oil-air: Self-cooled, no active cooling
48 MVA	Stage one: Directed oil pumps with fans
64 MVA	Stage two: Additional directed oil pumps with fans

Seasonal variations. As with generators, transformer ratings can vary with the ambient temperature, which affects the ability of the transformer to dissipate its internal heat. Therefore, the transformer's summer and winter ratings may be different, and must be considered in the seasonal power flow studies.

Emergency rating. Some utilities also include a transformer's **emergency rating** in their model, which is the power the transformer can handle for a short, specific time, such as 30 minutes. The emergency rating is designated as **RATEB**. However, the utility may elect to model only the continuous rating, in which case **RATEA** = **RATEB**.

Tertiary winding rating. The tertiary winding, if present, is rated as **RATEC**.

Per-unit Values

The transformer manufacturer specifies its own values for S_{base}, V_{base}, and impedance, Z, which is expressed as a percent at the specified S_{base}. **Therefore, it's critical to recalculate the transformer impedance per-unit values to the S_{base} and V_{base} you are using in your models, such as 100 MVA and 230 kV.** (See Appendix 1. "Per-unit Calculations and Expressions," for more information.)

> You must recalculate the transformer impedance per-unit values to the S_{base} and V_{base} you are using in your models, such as 100 MVA and 230 kV.

Transformer Cooling

During periods of light loading, a transformer can often dissipate enough heat through the natural circulation of the oil through the windings and radiator, especially if the ambient temperatures are low enough. As the load on the transformer increases, thermostats will start oil pumps to assist the natural circulation flow, and then switch on cooling fans, sometimes in stages, to increase the heat dissipation even further.

The nameplate in Figure 21 shows three continuous ratings that are a function of the cooling stages.

ANSI designation	IEC[4] designation	Description
OA	ONAN	Oil-air cooled (self-cooled)
FA	ONAF	Forced-air cooled
OA/FA/FA	ONAN/ONAF/ONAF	Oil-air cooled (self-cooled), followed by two stages of forced-air cooling (fans)
OA/FA/FOA	ONAN/ONAF/OFAF	Oil-air cooled (self-cooled), followed by one stage of forced-air cooling (fans), followed by 1 stage of forced oil (oil pumps)
OA/FOA	ONAF/ODAF	Oil-air cooled (self-cooled), followed by one stage of directed oil flow pumps (with fans)
OA/FOA/FOA	ONAF/ODAF/ODAF	Oil-air cooled (self-cooled), followed by two stages of directed oil flow pumps (with fans)
FOA	OFAF	Forced oil/air cooled (with fans) rating only—no self-cooled rating
FOW	OFWF	Forced oil / water cooled rating only (oil / water heat exchanger with oil and water pumps)—no self-cooled rating
FOA	ODAF	Forced oil / air cooled rating only with directed oil flow pumps and fans—no self-cooled rating
FOW	ODWF	Forced oil / water cooled rating only (oil / water heat exchanger with directed oil flow pumps and water pumps)— no self-cooled rating

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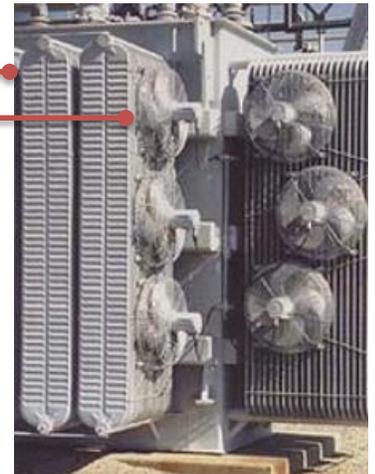
[4] International Electrotechnical Commission

Appendix 1. Per-unit Calculations and Expressions

Because the planner is dealing with so many different voltages across the transmission system, the convention has been to use the per-unit system that normalizes **power, voltage, current, impedance, and admittance** to a given base unit quantity. This simplifies calculations because the quantities are the same regardless of the voltage level. Of these five parameters, only two are independent—usually **power** and **voltage**—and from these, we can calculate the other three.
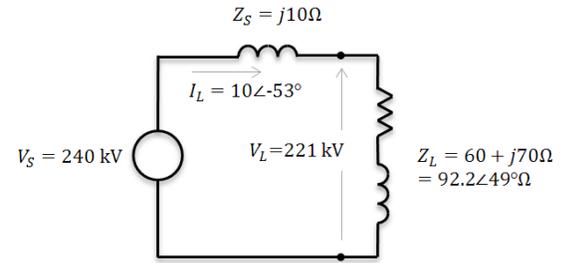
Figure 22 - A simple circuit with actual voltage, current, and impedance values.

Example

First we select (or are given) our per-unit base values for **power**, S_{base}, and **voltage**, V_{base}:

S_{base} = 100 MVA

V_{base} = 230 kV

From these two values, we can calculate the **base impedance**, Z_{base}:

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(230\ kV)^2}{100\ MVA} = 529\ \Omega$$

In the circuit above, the source voltage, V_s, is 240 kV, the source impedance, Z_S, is $j10\Omega$ (we're ignoring the very small impedance of the conductor), and the impedance of the load, Z_L, is:

$$Z_L = 60 + j70 = 92.2\angle 49°\Omega$$

Therefore, the total impedance of the circuit is:

$$Z_T = Z_S + Z_L = j10 + (60 + j70) = 60 + j80\ \Omega = 100\angle 53°\Omega \text{ (See Figure 23)}$$

The load current through the circuit is:

$$I_L = \frac{V_S}{Z_T} = \frac{240\ kV\angle 0°}{100\angle 53°} = 2400\ A\ \angle\text{-}53°$$

The voltage at the load is:

$$V_L = V_S \frac{Z_L}{Z_T} = 240\ kV \left(\frac{92.2}{100}\right) = 221\ kV$$

Note: X_L is j70 Ω. Fig. 23 will be changed in next revision so that green line is two parts: X_L and X_S, or redefine it as X_T.

Now we can express the circuit parameters in **per-unit** as:

Voltage at the source: $V_{S\ p.u.} = \frac{V_S}{V_{base}} = \frac{240\ kV}{230\ kV}$
$= 1.04$ p.u. (V)

Voltage at the load: $V_{L\ p.u.} = \frac{V_L}{V_{base}} = \frac{221\ kV}{230\ kV} = 0.96$ p.u. (V)

Impedance of the entire circuit: $Z_{T\ p.u.} = \frac{Z_T}{Z_{base}} = \frac{100}{529} = 0.198$ p.u. (Ω)

The real power dissipated at the load is:

$$P_L = I^2 R_L = (2400^2) \cdot (60) = 346\ MW$$

And in per-unit as: $P_{L\ p.u.} = \frac{P_L}{S_{base}} = \frac{346\ MW}{100\ MVA}$
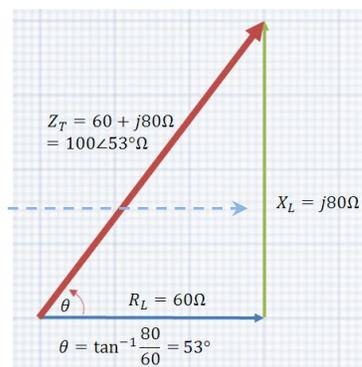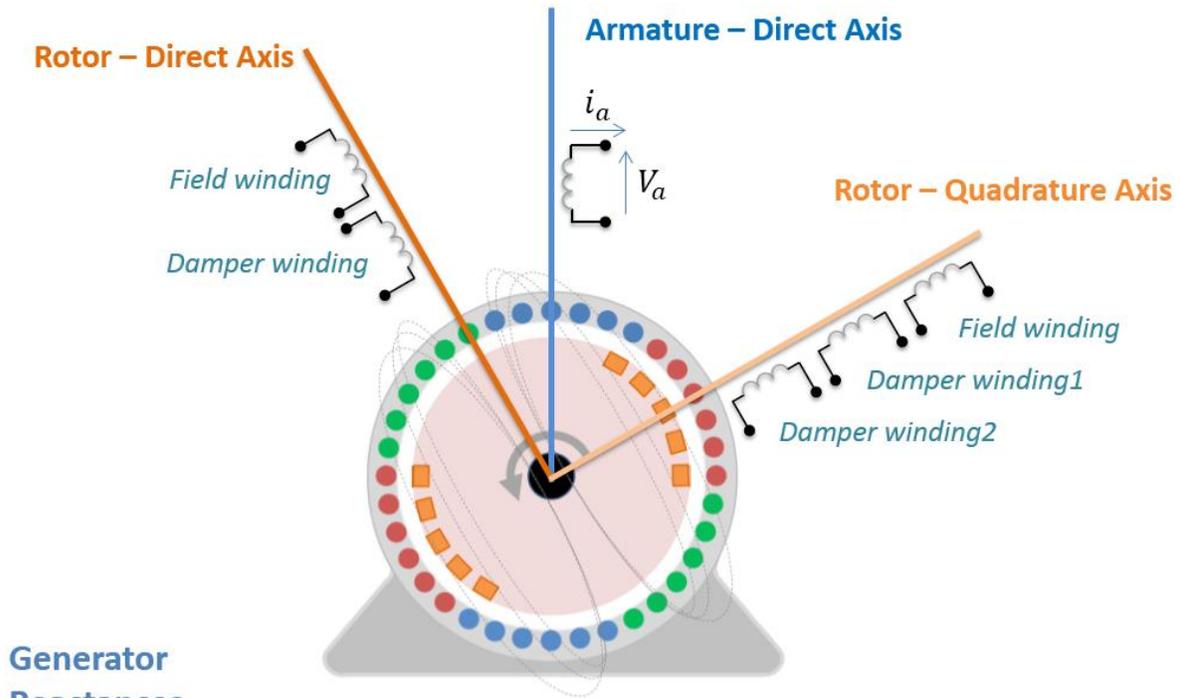$= 3.46$ p.u. (W)

Figure 23 - Converting the circuit impedance from R+jX to vector form.

Appendix 2.   The Generator Model

For a detailed explanation of the Generator Model, please see the **Generator Specifications Reference Document**.

Rotor – Direct Axis

Armature – Direct Axis

i_a

Field winding

V_a

Rotor – Quadrature Axis

Damper winding

Field winding

Damper winding1

Damper winding2

Generator Reactances

Round Rotor

$$X_d'' = X_q''$$

$$X_d, X_d', X_d''$$

$$X_q, X_q', X_q''$$

Appendix 3.   The Transformer Model

Introduction

The model of a two- or three-winding three-phase transformer include the impedance and admittance of each winding (primary, secondary, and tertiary), where **resistance and reactance are elements of impedance**, while **conductance and susceptance are elements of admittance**. These elements can be calculated using transformer settings, nameplate data, and test data.

The Transformer Equivalent Model

There are several equivalent circuit models for the "non-ideal"[5] transformer as shown in Figure 24. The resistance, R, is the resistance of the copper winding. The reactance, X, accounts for the flux leakage, which can also be described as a small amount of flux that travels thought the air outside the magnetic core path. The magnetizing conductance, G, represents the core loss of the magnetic core material due to hysteresis. The parallel inductance, B, accounts for the finite permeability of the magnetic core.
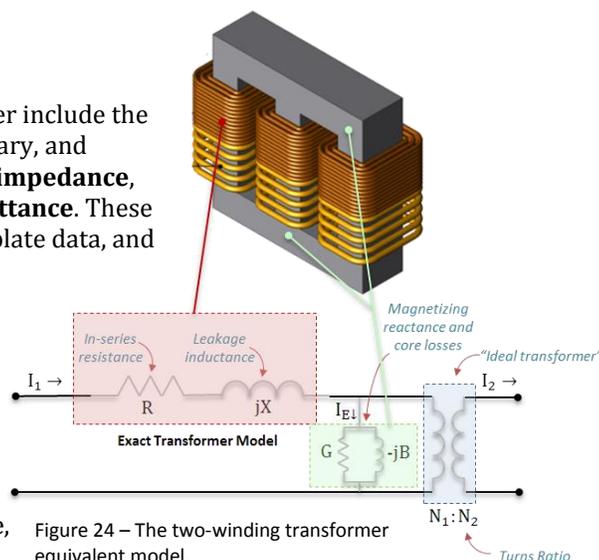

Figure 24 – The two-winding transformer equivalent model.

Models for 3-Winding Transformers

Three winding transformers are commonly modeled as a "Star" or "T" model in the power flow program. This results in an artificial node (or bus) in the center of the mo In order to calculate the three parameters of the Star model, the formulas below can I used. The manufacturer of the transformer will provide the three impedance measurements used in the formula. Once the Star model parameters are known they be entered into the power flow program. If the user desires not to use the Star model most power engineering text books provide the formulas to convert a Star model into Delta model. The Delta model does not require the artificial node in the center of the model. It can also result in negative impedances which can be difficult for some powe flow programs to solve and therefore is not recommended.
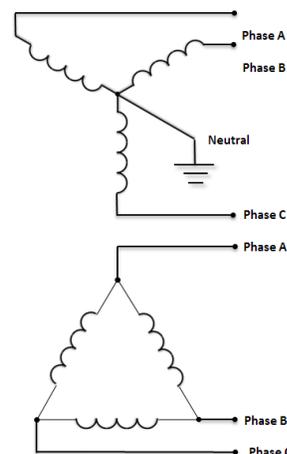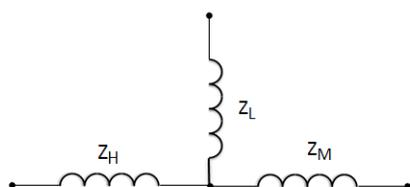

Figure 25 - It's possible to convert the "T" model into a Delta model (bottom), but the lack of an artificial center node may result in negative impedances that can be difficult for some power flow programs to solve.

$$Z_H = \frac{1}{2}\left[Z_{H+L} + Z_{H+M} - Z_{M+L}\right]$$

$$Z_M = \frac{1}{2}\left[Z_{H+M} + Z_{M+L} - Z_{H+L}\right]$$

$$Z_L = \frac{1}{2}\left[Z_{H+L} + Z_{M+L} - Z_{H+M}\right]$$

Figure 26 - Three-winding transformer modeled as a "star" or "T."

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[5] The "non-ideal" transformer comprises an "ideal" transformer plus the associated series impedances and shunt reactances that reflect physical reality.